NASA LANGLEY SEMI-ANNUAL STATUS REPORT
NAG 1–133 (3/1/81 – 8/31/81)

DEVELOPMENT OF AN ANALYTICAL TECHNIQUE FOR THE OPTIMIZATION OF JET ENGINE AND DUCT ACOUSTIC LINERS

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ABSTRACT

This report summarizes the work performed during the first six months of the NASA LANGLEY research program (Grant Number NAG 1-133) entitled "Development of an Analytical Technique for the Optimization of Jet Engine and Duct Acoustic Liners". Contained in this report is a brief summary of the development of the special integral representation of the external solutions of the Helmholtz equation which forms the basis for the analytical method developed under this contract. A detailed description of the new analytical technique for the generation of the optimum acoustic admittance for an arbitrary axisymmetric body is also presented along with some numerical procedures and some preliminary results for a straight duct.

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I. INTRODUCTION

The object of this research project is the development of an analytical technique which is capable of generating an optimum admittance distribution for a duct liner for maximum sound suppression. This analytical technique should yield this optimum distribution without iteration or the need for the calculation of many separate solutions. It is based upon a special integral representation of the external solutions of the Helmholtz equation previously developed here at Georgia Tech. The particulars of this method are presented in Section II.

The new analytical method itself is presented in Section III and some of the numerical procedures used in implementing the method are presented in Section IV. Briefly, the method entails the use of simple source solutions on the admittance surface of the body which are summed, using the linear superposition theorem for solutions of linear equations, to generate a general solution over the liner surface of the body. This general solution, is then substituted into the power equation and subsequently optimized with respect to the complex coupling constants, used in the linear superposition for the general solution, for maximum power lost to the liner surface.

The independent simple source solutions required for this method can be gotten by only solving the problem once due to the special form that the integral equation technique assumes when certain classes of simple boundary conditions are applied. This is gone into in more detail in Section IV which deals with numerical procedures.

II. BACKGROUND

In previous research conducted for the Air Force Office of Scientific Research[1,2] a special integral representation of the external solutions of the Helmholtz equation

$$\nabla^2\varphi + k^2\varphi = 0 \tag{1}$$

where k is the wave number and φ is the acoustic potential, was developed.[3] This integral formulation is special because unlike the straight forward formulation of the problem[4] it can generate unique solutions at all wave numbers. In subsequent research, the formulation and computer codes were specialized for axisymmetric bodies[5] but retained the capability of generating solutions for tangential acoustic modes greater than zero. It is this cylindrically symmetric formulation of the acoustic radiation problem that is used in this research. For the sake of completeness and to help define some of the nomenclature used in subsequent sections, the highlights of this development are presented below.

The classical integral representation of the external solutions of the Helmholtz equation is presented below[4] where S represents the surface of the body, the point Q is on the body, the point P lies outside the body, $\frac{\partial}{\partial n_q}$ represents the normal derivative $\nabla_q \cdot \vec{n}_q$ (where $\vec{n}_q$ is the unit outward normal to the body at the point Q), and G(P,Q) is any fundamental solution of the Helmholtz equation which satisfies the Sommerfeld radiation conditions at infinity.

$$\int\int_S \left\{ \varphi(Q) \frac{\partial G(P,Q)}{\partial n_q} - G(P,Q) \frac{\partial \varphi(Q)}{\partial n_q} \right\} dS_q = 4\pi\varphi(P) \tag{2}$$

The simplist form that G(P,Q) can take for this problem is the free space Green's function which is

$$G(P,Q) = \frac{e^{ikr(P,Q)}}{r(P,Q)} \tag{3}$$

where r(P,Q) represents the distance between the points P and Q (See Fig. 1.).

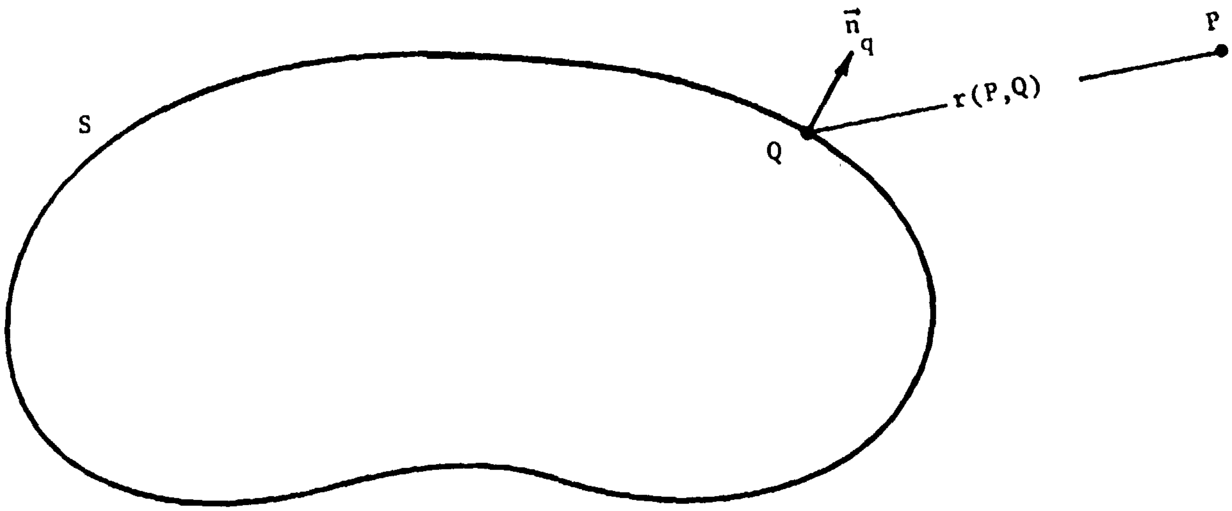

Figure 1. Body showing Q and P points and the distance between them r(P,Q).

From Eqn. (2) it follows that if the acoustic potential φ and the normal acoustic velocity $\frac{\partial\varphi}{\partial n}$ are known on the surface of the body, (i.e., at the Q points) then the acoustic potential may be calculated anywhere in the field outside the body (i.e., at any P point). A similar equation can be developed for the normal acoustic velocity with an arbitrary normal specified in the field and is presented below.

$$\int\int_S \left\{ \varphi(Q) \frac{\partial^2 G(P,Q)}{\partial n_p \, \partial n_q} - \frac{\partial G(P,Q)}{\partial n_p} \frac{\partial\varphi(Q)}{\partial n_q} \right\} dS_q = 4\pi \frac{\partial\varphi(P)}{\partial n_p} \tag{4}$$

Thus, if the acoustic quantities are known on the body, they may be calculated anywhere in the field.

To obtain the acoustic quantitites on the surface of the body, equations must be developed that contain only surface quantities. To do this, we let the field point P approach the surface of the body; then, taking the proper limits we obtain

$$\iint_S \left\{ \varphi(Q) \frac{\partial G(P,Q)}{\partial n_q} - G(P,Q) \frac{\partial \varphi(Q)}{\partial n_q} \right\} dS_q = 2\pi\varphi(P) \tag{5}$$

from Eqn. (2). With the proper boundary conditions (i.e., φ, $\partial\varphi/\partial n$ or $Y = \frac{\partial\varphi}{\partial n}/\varphi$) known at each point on the surface of the body Eqn. (5) may be solved for the unknown acoustic quantities. From here on, Y shall be referred to as the effective acoustic admittance; also, it is related to the specific acoustic admittance β by the relationship

$$Y = -ik\beta \tag{6}$$

where β is defined with an inward normal and Y is defined with an outward normal.

The problem with Eqn. (5) is that it does not yield unique solutions for all wave numbers. This can be traced to the internal acoustic eigenvalue problem which when formulated in the same way is governed by an equation similar to Eqn. (5) except that the sign of the R. H. S. is negative. This being the case, it is found that Eqn. (5) does not yield unique solutions at the eigenvalues (i.e., resonant wave numbers) of the internal acoustic problem. Various methods have been proposed in the literature for overcoming this problem[5-8] however, all of these methods have their problems. This is discussed in detail in Ref. 3.

To overcome this uniqueness problem, the method of Burton and Miller[8] was used as a starting point. They were able to solve external radiation problems in two dimensions; however, the extension to three dimensions required some new mathematical identities before it could be made to work as the 3-D formulation contained a strongly singular integral. Briefly, the method consisted of solving a sum of equations (i.e., Eqn. (5) and the surface analog of Eqn. (4)) coupled with a complex coupling constant α .

$$\int_S\int \left\{ \varphi(Q) \frac{\partial G(P,Q)}{\partial n_q} - \frac{\partial \varphi(Q)}{\partial n_q} G(P,Q) \right\} dS_q$$

$$+ \alpha \int_S\int \left\{ \varphi(Q) \frac{\partial^2 G(P,Q)}{\partial n_p \, \partial n_q} - \frac{\partial \varphi(Q)}{\partial n_q} \frac{\partial G(P,Q)}{\partial n_p} \right\} dS_q$$

$$= 2\pi \left\{ \varphi(P) + \alpha \frac{\partial \varphi(P)}{\partial n_p} \right\} \qquad (7)$$

Burton and Miller were able to show that Eqn. (7) always had the unique, correct solution if α were chosen properly; specifically:

$$\text{Im } (\alpha) \neq 0 \text{ when k is real or imaginary}$$

$$\text{Im } (\alpha) = 0 \text{ when k is a complex number} \qquad (8)$$

The problem arises in three dimensions that the first term of the second integral is strongly singular and cannot be directly integrated; that is:

$$\int_S\int \varphi(Q) \frac{\partial^2 G(P,Q)}{\partial n_p \, \partial n_q} \, dS_q \rightarrow \mathcal{O}\left(\frac{1}{r(P,Q)} \right) \qquad (9)$$

which is singular as $Q \rightarrow P$ on the surface of the body.

Stallybrass[9] was able to show that this integral is equivalent to

$$\int_S\int \varphi(Q) \, (n_p \cdot n_q) \, \nabla_p \cdot \nabla_q \, G(P,Q) \, dS_q$$

$$+ \int_S\int \varphi(Q) \, (n_p \times n_q) \cdot (\nabla_p \times \nabla_q \, G(P,Q)) \, dS_q \qquad (10)$$

$$- \int_S\int \varphi(Q) \, n_q \cdot \left\{ \nabla_q \times (n_p \times \nabla_p \, G(P,Q)) \right\} dS_q$$

and that the last term, which contains the singular component of the integral in Eqn. (9), can be represented as

$$\int\!\!\int_S (n_q \times \nabla_q \varphi(Q)) \cdot (n_p \times \nabla_p G(P,Q))\, dS_q \qquad (11)$$

which is regular. Although the singular integral has been regularized, this form is not suitable for numerical calculations as it contains tangential derivatives of the acoustic potential on the surface of the body.

After some manipulation, it can be shown that this integral (See Eqn. (11).) can be rewritten as

$$\int\!\!\int_S \left\{ \varphi(Q) - \varphi(P) \right\} n_q \cdot \nabla_q \times (n_p \times \nabla_p G(P,Q))\, dS_q \qquad (12)$$

which presents no computational difficulties. Thus, the singular integral has been shown to have a regular representation which can be easily integrated numerically. The remaining practical problem was now the specification of a reasonable value for α subject to the constraints in Eqn. (8). Since no analytical method of determining the value of α could be found, its specification is the result of computational considerations. Specifically, it can be shown that the most significant term of the first integral in Eqn. (7) is proportional to the wave number k and that the most significant term of the second integral increases as k^2. So to keep the two integrals in Eqn. (7) in balance numerically as the wave number is increased, the complex coupling constant α is chosen to be

$$\alpha = i/k \qquad (13)$$

It is shown in Ref. 10 though the use of many examples, that this is indeed the optimum value of σ from a computational point of view.

Having developed the general three dimensional equations, the specialization of these equations for axisymmetric bodies is straight forward[10] and therefore will not be repeated here. Efficient computer codes have been written for the solution of these equations and the results of these computer codes have been compared with both theoretical "exact" solutions in Ref. 10 and with experimental results in Ref. 11. In both cases, very good agreement was observed.

III. THE ANALYTICAL METHOD

The object of this research project is the development of an analytical method and attendant computer programs for the determination of the optimum admittance distribution of a liner for maximum sound suppression for a specific body and acoustic excitation without iteration. This method will contain two advances over previous methods[12] for finding the optimum admittance for liners: 1) this new method will not require iteration in order to generate the optimum solution of the problem and 2) the optimum solution generated will yield a pointwise continuous distribution of admittance values which should demonstrate better sound suppression than optimum constant or segmented liners. To generate the point source solutions necessary for this method to work, the cylindrically symmetric form of the theory developed in the previous section[10] is used as most bodies of interest (e. g. jet engine inlets and straight circular ducts) are axisymmetric. This will be gone into in greater detail in subsequent sections.

Since the objective of this research is to minimize the energy radiated from a body under specific acoustic excitation through the use of an acoustic liner, the problem can be turned around so that the objective is to maximize the acoustic energy absorbed by the liner. Contained in the problem statement are the implicit assumptions that: 1) the placement of the liner is fixed; 2) the specific acoustic excitation is fixed by the assumption of a distribution of acoustic potential (i.e., the same as the specification of the acoustic pressure), and 3) the liner can be represented by an acoustic admittance (i.e.,it is a surface of local reaction).[13] This being the case, the acoustic energy absorbed by the liner can be represented as

$$E = - \int\int_{S_\ell} \bar{\rho} \bar{c}\, k\, Y^I \left| \phi \right|^2 dS \qquad (14)$$

where the subscript ℓ refers to the liner surface and the superscript I denotes the

"imaginary part of." Using the definition $Y = \frac{\frac{\partial\varphi}{\partial n}}{\varphi}$ (See Eqn. (6).) this can be written as

$$E \propto \iint_{S_\ell} \left\{ \frac{\partial\varphi}{\partial n}^R \varphi^I - \frac{\partial\varphi}{\partial n}^I \varphi^R \right\} dS \tag{15}$$

where the superscript R denotes the "real part of" and all values are assumed to be R. M. S.

The analytical optimization procedure entails the maximization of E as defined in Eqn. (15) where the acoustic quantities are represented in terms of a general solution consisting of a combination of simple source solutions on the surface of the body. The development of this general solution is presented below wherein the body of interest is assumed to have three distinct regions on its surface (See Fig. 2.).

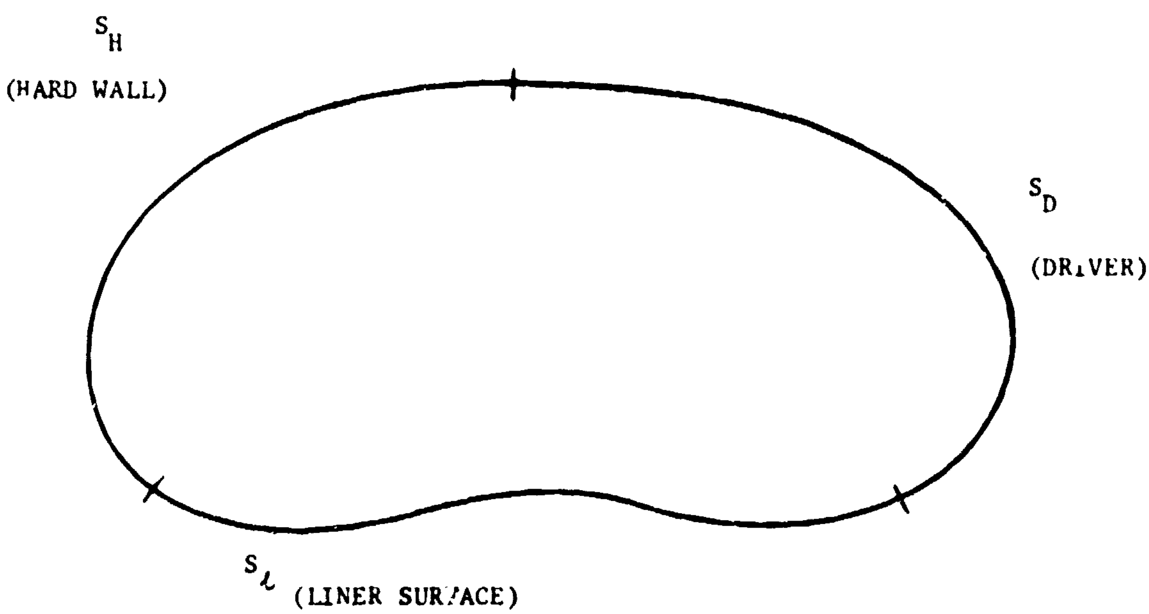

Figure 2. The three distinct types of regions on the body.

These regions do not necessarily have to be contiguous however for the sake of clarity they are presented as such here.

To form the general solution we first must consider the effect of the driver surface (s). To do this we solve for the acoustic quantities on the surface of the body subject to the boundary conditions

$$
\begin{aligned}
\varphi(Q) &= \tilde{\varphi}_D(Q) && \text{on } S_D \\
\frac{\partial\varphi}{\partial n}(Q) &= 0 && \text{on } S_H \text{ and } S_\ell
\end{aligned}
\tag{16}
$$

where $\tilde{\varphi}_D(Q)$ is some specified function of the acoustic potential on the driver. Solving this problem we obtain the driver solution; that is:

$$
\begin{aligned}
&\frac{\partial\varphi_D}{\partial n}(Q) && \text{on } S_D \\
&\varphi_D(Q) && \text{on } S_H \text{ and } S_\ell
\end{aligned}
\tag{17}
$$

Next, the liner surface (s) is divided up into N finite regions as in Fig. 3.

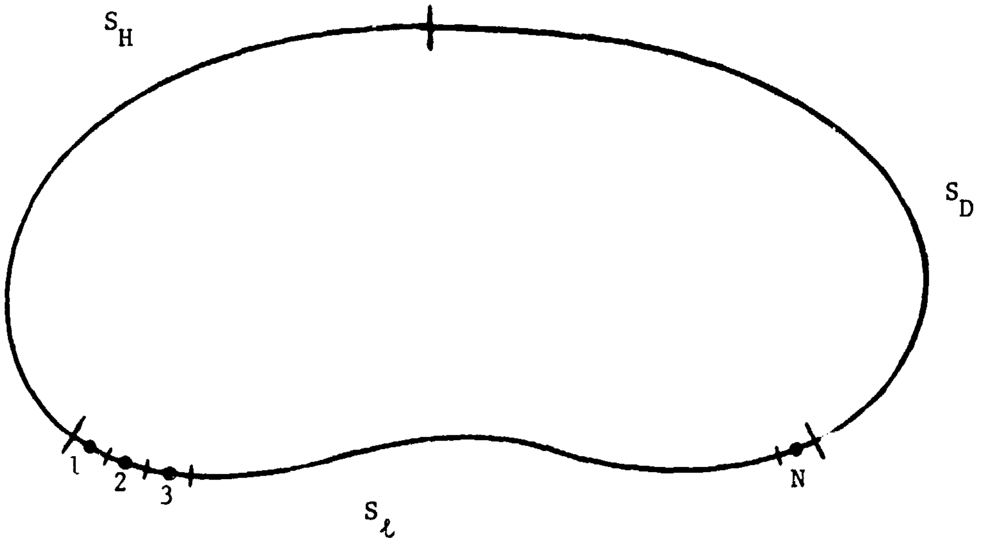

Figure 3. Liner surface divided into N finite regions.

Then N independent solutions are generated which represent the effect of N simple acoustic velocity sources on the liner using the boundary conditions given below.

$$
\left.
\begin{array}{ll}
\varphi(Q) = 0 & \text{on } S_D \\
\frac{\partial\varphi}{\partial n}(Q) = 0 & \text{on } S_H \\
\left.\begin{array}{l}
\frac{\partial\varphi}{\partial n}(Q_j) = 1 \quad j = 1,\ldots,N \\
\frac{\partial\varphi}{\partial n}(Q_i) = 0 \quad i \neq j
\end{array}\right\} & \text{on } S_\ell
\end{array}
\right.
\qquad (18)
$$

The N solutions thus generated are given by

$$
\begin{array}{ll}
\frac{\partial\varphi}{\partial n}{}_j(Q) & \text{on } S_D \\
\varphi_j(Q) & \text{on } S_H \\
\varphi_j(Q) & \text{on } S_\ell
\end{array}
\qquad (19)
$$

If we now sum these solutions multiplied by some arbitrary coupling constants designated by a_j, which we can do as the problem is linear, we generate a general solution which has the form

$$
\left.
\begin{array}{l}
\varphi(Q) = \tilde{\varphi}(Q) \\
\frac{\partial\varphi}{\partial n}(Q) = \frac{\partial\varphi_D}{\partial n}(Q) + \sum_{j=1}^{N} a_j \frac{\partial\varphi_j}{\partial n}(Q)
\end{array}
\right\} \text{on } S_D \qquad (20)
$$

$$
\left.
\begin{array}{l}
\varphi(Q) = \varphi_D(Q) + \sum_{j=1}^{N} a_j \varphi_j(Q) \\
\frac{\partial\varphi}{\partial n}(Q) = 0
\end{array}
\right\} \text{on } S_H \qquad (21)
$$

$$\varphi(Q) = \varphi_D(Q) + \sum_{j=1}^{N} a_j \varphi_j(Q)$$

$$\frac{\partial\varphi}{\partial n}(Q_j) = a_j \qquad j = 1,\ldots,N \qquad \text{on } S_\ell \tag{22}$$

$$\frac{\partial\varphi}{\partial n}(Q_j) = 0 \qquad i \neq j$$

It will be noted here that the above solution has some interesting properties in that the acoustic potential on the driver surface (See Eqn. (20).) and the normal acoustic velocity on the hard walled surface (See Eqn. (21).) are not dependent upon the choice of the coupling constants a_j. Also, strictly speaking all possible values of the effective admittance Y are not possible on the liner surface. To demonstrate this, let us look at the point j = 1 on the liner surface where

$$Y(Q_1) = \frac{\partial\varphi(Q_1)}{\partial n} \Big/ \varphi(Q_1) = \frac{a_1}{\varphi_D(Q_1) + \sum_{i=1}^{N} a_i \varphi_i(Q_1)} \tag{23}$$

Solving for a_1, we obtain

$$a_1 = \frac{Y(Q_1) \sum_{i=2}^{N} a_i \varphi_i(Q_1)}{1 - Y(Q_1)\varphi_1(Q_1)} \tag{24}$$

where it can be seen that if we want $Y(Q_1) = \frac{1}{\varphi_1(Q_1)}$ we must have $a_1 \to \infty$. Thus, we cannot generate the solution where the effective admittance $Y(Q_j) = \frac{1}{\varphi_j(Q_j)}$ with finite values for the complex coupling constants a_j.

If we now substitute the expressions for the acoustic quantities on the liner surface (See Eqn. (22).) into the equation for the power lost to the admittance surface Eqn. (15) and treat the surface integral as a sum we obtain

$$\sum_{j=1}^{N} \Big\{ a_j^R \Big[\varphi_D^I (Q_j) + \sum_{i=1}^{N} \big[a_i^R \varphi_i^I (Q_j) + a_i^I \varphi_i^R (Q_j) \big] \Big]$$

$$- a_j^I \Big[\varphi_D^R (Q_j) + \sum_{i=1}^{N} \big[a_i^R \varphi_i^R (Q_j) - a_i^I \varphi_i^I (Q_j) \big] \Big] \Big\} \Delta S_\ell (Q_j) \tag{25}$$

If we now want to maximize the power lost to the admittance surface with respect to the real and imaginary parts of the complex coupling constants we must take the derivatives of Eqn. (25) with respect to the constants:

$$\frac{\partial}{\partial a_j^R} \Big\{ \text{Eqn. } (25) \Big\} = 0$$

$$j = 1, \ldots, N \tag{26}$$

$$\frac{\partial}{\partial a_j^I} \Big\{ \text{Eqn. } (25) \Big\} = 0$$

and set them equal to zero. Doing this we get

$$\Big\{ \varphi_D^I (Q_j) + \sum_{i=1}^{N} \big[a_i^R \varphi_i^I (Q_j) + a_i^I \varphi_i^R (Q_j) \big] \Big\} \Delta S_\ell (Q_j)$$

$$+ \sum_{i=1}^{N} \big[a_i^R \varphi_j^I (Q_i) - a_i^I \varphi_j^R (Q_i) \big] \Delta S_\ell (Q_i) = 0$$

$$j = 1, \ldots, N \tag{27}$$

$$\left\{-\varphi_D^R(Q_j) - \sum_{i=1}^{N}\left[a_i^R\,\varphi_i^R(Q_j) - a_i^I\,\varphi_i^I(Q_j)\right]\right\}\Delta s_\ell(Q_j)$$

$$+\sum_{i=1}^{N}\left[a_i^R\,\varphi_j^R(Q_i) + a_i^I\,\varphi_j^I(Q_i)\right]\Big\}\Delta s_\ell(Q_i) = 0$$

which upon rearrangement becomes:

$$\sum_{i=1}^{N}\left[a_i^R\,\varphi_i^I(Q_j) + a_i^I\,\varphi_i^R(Q_j)\right]\Delta s_\ell(Q_j)$$

$$+\sum_{i=1}^{N}\left[a_i^R\,\varphi_j^I(Q_i) - a_i^I\,\varphi_j^I(Q_i)\right]\Delta s_\ell(Q_i) = -\varphi_D^I(Q_j)\,\Delta s_\ell(Q_j)$$

$$j = 1,\ldots,N \tag{28}$$

$$-\sum_{i=1}^{N}\left[a_i^R\,\varphi_i^R(Q_j) - a_i^I\,\varphi_i^I(Q_j)\right]\Delta s_\ell(Q_j)$$

$$+\sum_{i=1}^{N}\left[a_i^R\,\varphi_j^R(Q_j) + a_i^I\,\varphi_j^I(Q_i)\right]\Delta s_\ell(Q_i) = \varphi_D^R(Q_j)\,\Delta s_\ell(Q_j)$$

If we now define the complex conjugates of the original variables as:

$$\hat{a}_j = a_j^R - ia_j^I$$

$$\hat{\varphi}_j = \varphi_j^R - i\varphi_j^I \tag{29}$$

$$\hat{\varphi}_D = \varphi_D^R - i\varphi_D^I$$

the two sets of real equations in Eqn. (28) can be reformulated as one complex set of equations given by

$$- \sum_{i=1}^{N} \left[\hat{a}_i \, \hat{\varphi}_i(Q_j)\right] \Delta S_\ell(q_j) + \sum_{i=1}^{N} \left[\hat{a}_i \, \varphi_j(Q_i)\right] \Delta S_\ell(Q_i) \tag{30}$$

$$= \hat{\varphi}_D(Q_j) \, \Delta S_\ell(Q_j) \qquad j = 1,\ldots,N$$

where the $\hat{a}_j$ are now the unknowns. As can be seen, Eqn. (30) represents N complex equations in N complex unknowns and can therefore be solved by straight forward numerical means. Once the optimum values of the complex coupling constants are calculated, the optimum surface distributions of the acoustic quantities may be found through the use of Eqns. (20) - (22). Then the power radiated to the field may be found using Eqns. (2) and (4) to calculate the acoustic quantities in the field on an imaginary sphere surrounding the body and then using Eqn. (14) to calculate the power.

IV. NUMERICAL PROCEDURES

The method outlined in the previous section requires not only the generation of a driver solution for the body of interest but also the generation of many simple source solutions on the admittance surface. If each of these solutions had to be generated separately, the present method would be no more attractive from a computational stand point than the method of Ref. 12 where many separate solutions are also necessary to find the optimum conditions. Thus, a way had to be found to generate the required source solutions efficiently.

The main computational advantage of the present method can only be realized when the method is coupled with the integral solution technique set forth in the Section II. In solving Eqn. (7) for the surface quantities, the coefficients of the unknowns are placed in a matrix while the knowns (i.e., the boundary conditions) are collected into an inhomogeneous vector. Thus, each simple source solution requires the solution of a liner set of equations.

If the boundary conditions are chosen correctly, only the inhomogeneous vector changes and therefore the matrix of coefficients for the unknowns only has to be inverted once. A special matrix solving routine was then written to take advantage of this which solves a system of linear equations with multiple inhomogeneous vectors very efficiently. This being the case, the multiple simple source solutions and the driver solution necessary for the optimization method can be generated all at once using little more computing time than it takes to calculate the driver solution alone.

One of the potential problems that had to be checked for was if the integral solution technique was capable of generating simple source solutions. Normally when Eqn. (7) is discretized the non-zero boundary conditions (e.g., the potential on the driver surface) are specified on a number of successive points on the body. Since relatively large errors have been found to exist where boundary conditions change abruptly when using the

integral solution procedure, it was of concern that accurate source solutions might not be gotten using this method. This was checked by first generating a number of simple source solutions on a body and summing them, using the linear superposition principle, and then comparing this result with one generated specifying all the points on the body together. Excellent agreement was found between the two solutions generated in this way which was considered to be justification that accurate simple source solutions could indeed be calculated using the integral equation techniques

Once the computer program is written to generate the independent solutions necessary for the optimization procedure, the generation of the optimum admittance distribution on the body is straight forward. Substituting the independent solutions into Eqn. (30) another system of linear, complex equations is generated which can be solved by straight forward Gaussian elimination for the complex coupling constants (i.e., the a_j's) Having done this, the optimum admittance on the liner surface can be directly calculated from Eqn. (22).

V. SOME PRELIMINARY RESULTS

The test body being used for verification of the method is a straight duct with a rounded lip, an external wall thickness of 0.15a where a is the non-dimensional distance (i.e., the radius at the driver plane) and an L/a of 2.0 where L is the length of the duct (See Fig. 4.). Also, the liner surface is considered to run from a/2 to 3a/2 on the inner wall of the duct and the duct is terminated by an ellipse whose ratio of major to minor axis is 2.0. For the test case, plane wave input is assumed with an acoustic mode of M = (0,0) and a non-dimensional wave number of ka = 1.0. It will be noted here that this simple case is being used as a test case only and that more complicated cases, that is a true inlet shape at a higher wave number with a more complicated modal input, can easily be run without changing the computer codes; only the input files need to be changed.

For the numerical calculations, the straight duct is broken into 92 separate line segments along the body: 20 of these are on the driver and 25 are on the admittance surface. Also, in carrying out the numerical integrations necessary in the tangential direction (recall that a cylindrically symmetric formulation of the problem is used) a 32 point Gauss-Legendre formula is used.

Using the Georgia Tech CDC-CYBER 70/74, the generation of the driver solution and the 25 independent source solutions on the admittance surface required only 3 minutes of computing time. This compares favorably with the time required to calculate one single solution using the same body and number of points which takes ~ 2½ minutes. Once these have been gotten the next program requires only 30 seconds to calculate the complex coupling constants and the optimum solution.

The hard wall (or driver) solution radiates a power of P = 1.91 out of the duct where the power is calculated at the driver plane using

$$P = \iint_{S_D} \left(\frac{\partial\varphi^I}{\partial n} \varphi^R - \frac{\partial\varphi^R}{\partial n} \varphi^I \right)(ka)\,dS_D \qquad (31)$$

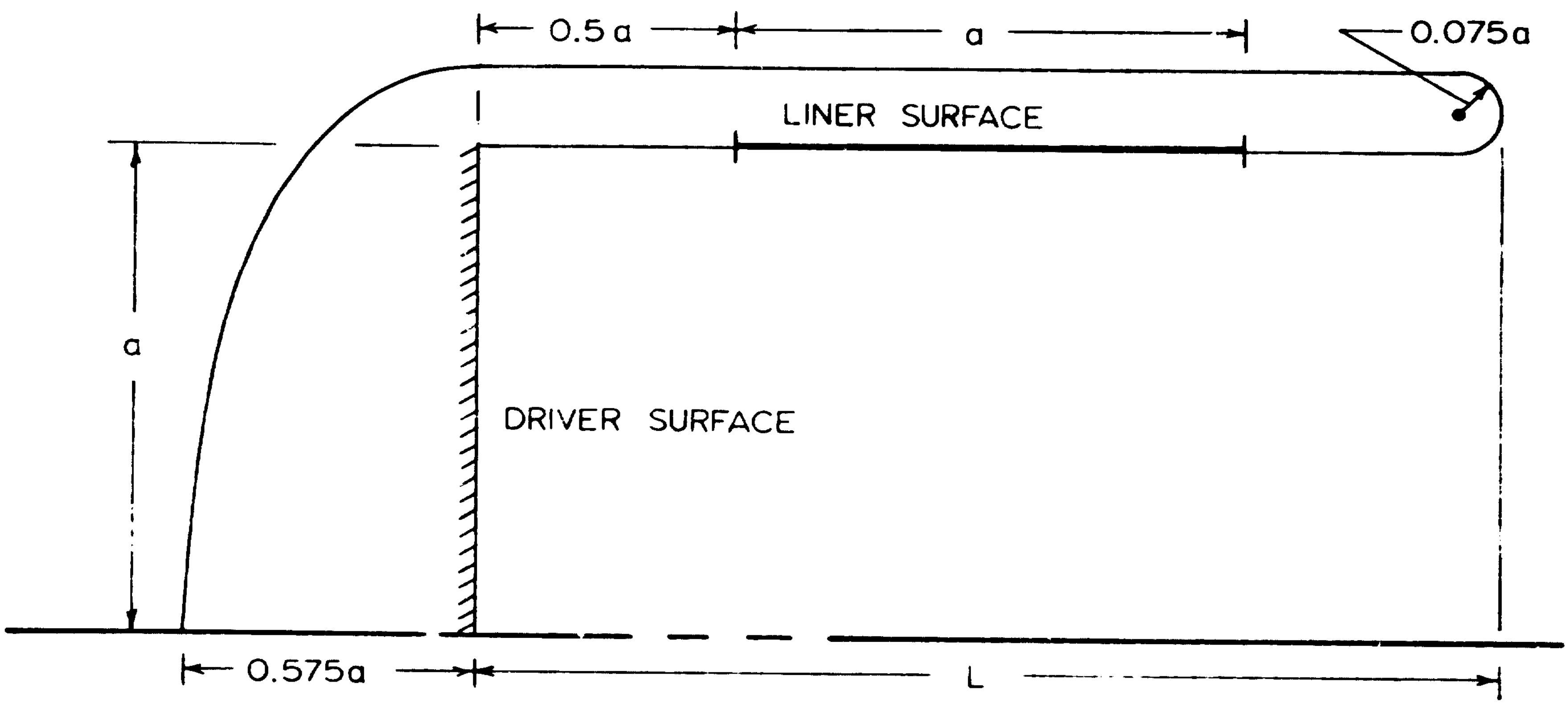

Figure 4. Straight Duct Geometry (L/a = 2.0).

where it will be noted that the wave number dependence has been kept (although in the present case ka = 1.0). This number should be used for comparison purposes.

The optimum admittance calculation for this case yielded a power lost to the admittance surface of P = 143.44. To get the power lost to the admittance surface Eqn. (31) was used except that the integration was performed over the admittance surface. To obtain the power radiated out of the duct, the power out of the driver plane must be recalculated as changing the admittance surface from a hard wall to an admittance distribution changes the driver. Doing this, it was found that the power out of the driver rose to P = 287.96 so that the power out of the duct was increased from P = 1.91 in the hard walled case to P = 144.52. As can be seen, this is not what we had hoped for but upon review it is exactly what we asked for; that is, find the solution (i.e., the admittance distribution on the liner surface) which yields the greatest power lost to the liner surface.

The above results show that just maximizing the power lost to the admittance surface does not necessarily minimize the power out of the duct. This is thought to be the result of the fact that the effect of the liner on the driver was not taken into account in the present formulation. This being the case, a reformulation of the problem has been accomplished which takes into account the effect the admittance on the liner surface has on the power output of the driver. This alternative formulation is presented in the next section.

VI. AN ALTERNATIVE FORMULATION

In the original formulation of the problem, the effect of changing the admittance on the liner surface on the power output of the driver was not taken into account. Since subsequent calculations have shown that the admittance on the liner surface can significantly effect the power output of the driver, an alternative formulation of the problem has been developed which includes the driver surface in the power calculation. In short this new formulation seeks to minimize the power out of the combination of the driver and admittance surfaces rather than simply maximizing the power lost to the liner surface. This formulation should yield the minimum power out of the duct rather than the maximum power lost to the admittance surface.

If we develop an equa' on for the power radiated out of the driver surface (similar to Eqn. (25) for the liner surface) we obtain

$$\sum_{j=K}^{M} \Bigg\{ \Bigg[\frac{\partial \varphi_D^R (Q_j)}{\partial n} + \sum_{i=1}^{N} \Bigg[a_j^R \frac{\partial \varphi_i^R (Q_j)}{\partial n} - a_i^I \frac{\partial \varphi_i^I (Q_j)}{\partial n} \Bigg] \Bigg] \varphi_D^I (Q_j)$$

$$- \Bigg[\frac{\partial \varphi_D^I (Q_j)}{\partial n} + \sum_{i=1}^{N} \Bigg[a_i^R \frac{\partial \varphi_i^I (Q_j)}{\partial n} + a_i^I \frac{\partial \varphi_i^R (Q_j)}{\partial n} \Bigg] \Bigg] \varphi_D^R (Q_j) \Bigg\} \Delta S_D (Q_j) \tag{32}$$

from Eqns. (15) and (20) where K and M are the beginning and ending points on the driver surface (recall the admittance surface goes from 1 to N). Carrying out the operation of Eqn. (26) on Eqn. (32) we obtain in complex notation

$$- \sum_{j=K}^{M} \Bigg\{ \frac{\partial \varphi_i (Q_j)}{\partial n} \hat{\varphi}_D (Q_j) \Bigg\} \Delta S_D (Q_j) = 0 \qquad i = 1, \ldots, N \tag{33}$$

Adding this to Eqn. (30) we obtain

$$- \sum_{i=1}^{N} \left[a_i \hat{\varphi}_i(Q_j) \right] \Delta S_{\ell}(Q_j) + \sum_{i=1}^{N} \left[\hat{a}_i \varphi_j (Q_i) \right] \Delta S_{\ell}(Q_i)$$

$$= \hat{\varphi}_D (Q_j) \Delta S_{\ell} (Q_j) + \sum_{i=K}^{M} \frac{\partial \varphi_i (Q_i)}{\partial n} \hat{\varphi}_D (Q_i) \Delta S_D(Q_i) \qquad (34)$$

$$j = 1, \ldots, N$$

which a set of N simultaneous linear equations for the $\hat{a}_j$'s. This reformulation of the problem is now being programmed.

VII. CHECK CASES

In order to check the results of the new method optimum constant liner results are necessary for the specific geometrys and wave numbers used in this study. To obtain these results, computer programs have been written and checked out which calculate the power radiated from an axisymmetric body with an acoustic liner. These programs are based on the integral equation technique presented in Section II and will employ the method of Ref. 12 which entails the calculation of many separate solutions. The optimum constant liner admittance will be calculated for each body and for each modal input (i. e., at each wave number) for which the optimum admittance calculation is run.

VIII. SUMMARY

During the first six months of this research project, the computer programs were written and checked out which are necessary to implement the new analytical method for the calculation of the optimum liner admittance for sound suppression in a duct. This analytical method was designed to maximize the power lost to the admittance surface which it accomplished very well. Unfortunately, since the effect of the liners admittance on the power output of the driver was not considered, this scheme did not optimize for the minimum power out of the duct. In fact, the sound power out of the duct was increased drastically over the hard walled case. Thus, a new theoretical method was developed which is designed to minimize he power out of the duct by considering both the power output of the driver and the power loss to the admittance surface. The computer programs are currently being modified to handle the extra terms this method requires.

To generate the required optimum admittace check cases, computer programs have been written and checked out which can calculate the power output of a duct under specific excitation conditions with a liner surface in the duct. These programs are currently being run to find the optimum constant admittance for sound suppression for each configuration of interest.

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